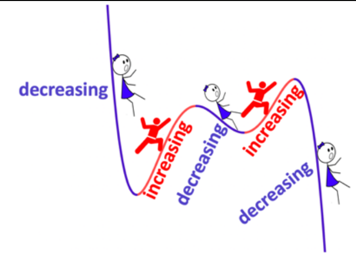
First Derivative $\frac{dy}{dx}$ (talks about slope)		Second Derivative $\frac{d^2y}{dx^2}$ (talks about concavity)	
Increasing and decreasing	Stationary/Turning Points (Max or Min)	Concave Up (convex) Concave Down (aka concave)	Points of Inflection

Increasing and decreasing



Increasing:
This is where the slope is positive. Imagine someone climbing up a hill.

solve $\frac{dy}{dx} > 0$

Decreasing:
This is where the slope is negative. Imagine someone sliding down a hill.

solve $\frac{dy}{dx} < 0$

Important: Remember that when we solve an inequality that is a quadratic or higher we must use the sign change test or graph! We cannot just guess the signs!

Stationary/Turning Points (Max or Min)

Max/Min:
This is where increasing changes to decreasing or vice versa. The slope is neither positive, nor negative, it is zero!

solve $\frac{dy}{dx} = 0$

To verify whether a max or min:

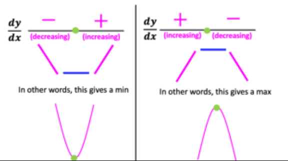
Way 1: Plug into $\frac{d^2y}{dx^2}$

Plug the x value found into $\frac{d^2y}{dx^2}$ and if

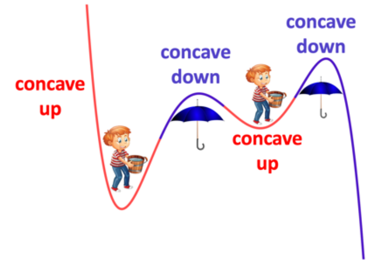
$$\frac{d^2y}{dx^2} > 0 \Longrightarrow \text{min}$$
$$\frac{d^2y}{dx^2} < 0 \Longrightarrow \text{max}$$

Way 2: Sign change number line test for $\frac{dy}{dx}$

We plug in an x value just below and just above the x value found into $\frac{dy}{dx}$. If $\frac{dy}{dx}$ changes sign from negative $(-)$ to $(+)$ then min and if $\frac{dy}{dx}$ changes from positive $(+)$ to negative $(-)$ then max.



Concave Up (convex) / Concave Down (aka concave)



Concave Up/Convex:
Imagine a bowl or the inside of a bucket. Concave up means the rainwater would be held by the bucket and hence held by the curve.

solve $\frac{d^2y}{dx^2} > 0$

Concave Down/Concave:
Imagine an upside-down bowl or the inside of umbrella. Concave down means the rainwater would roll off and hence would roll off the curve.

solve $\frac{d^2y}{dx^2} < 0$

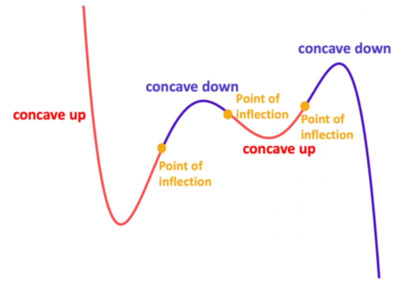
Important: Remember that when we solve an inequality that is a quadratic or higher we should use the sign change test or graph! We cannot just guess the signs!

Note: It should now make sense why we have the criteria

$\frac{d^2y}{dx^2} > 0 \Longrightarrow$ min (holds water)

$\frac{d^2y}{dx^2} < 0 \Longrightarrow$ max (spills water)

Points of Inflection



Points of inflection:
These are where concavity changes from concave up to down or vice versa. The concavity is neither positive, nor negative, it is zero!

solve $\frac{d^2y}{dx^2} = 0$

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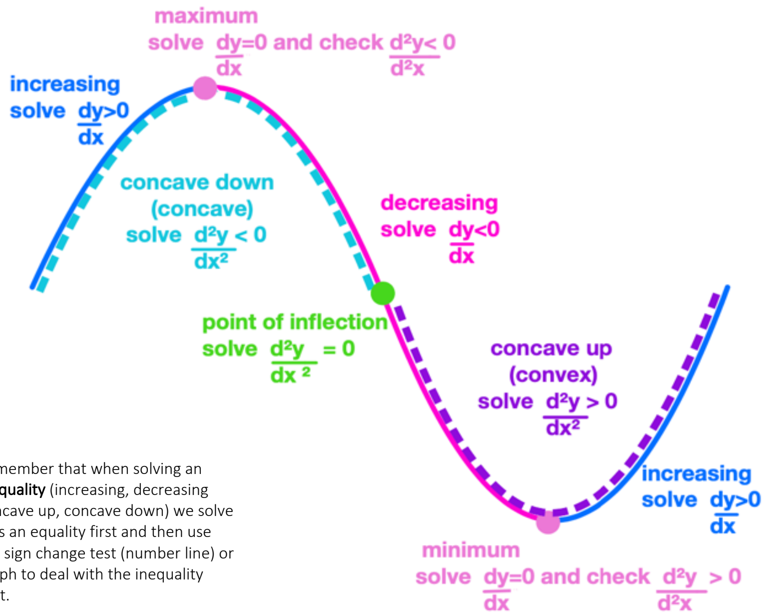
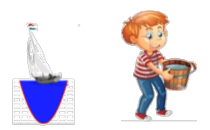
Summary

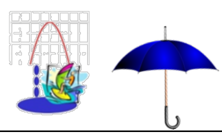
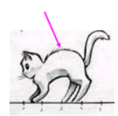


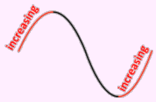
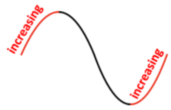
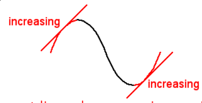
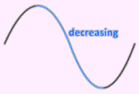
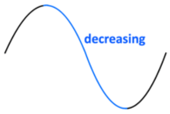
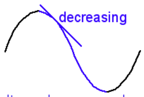
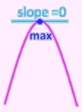
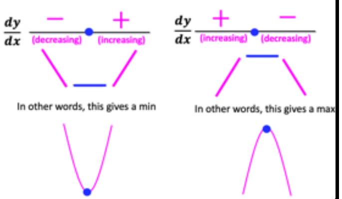
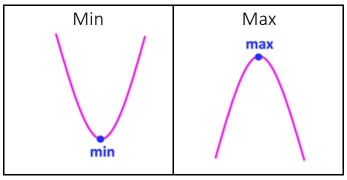
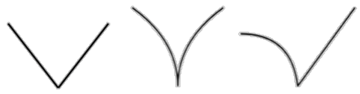
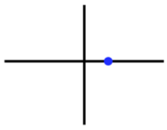
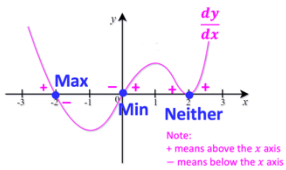
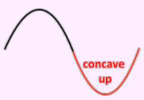
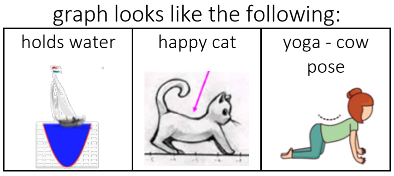
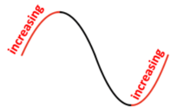
Remember that when solving an **inequality** (increasing, decreasing concave up, concave down) we solve it as an equality first and then use the sign change test (number line) or graph to deal with the inequality part.

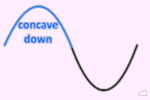
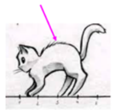
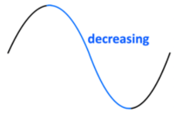
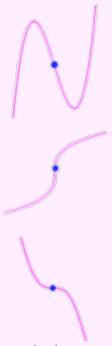
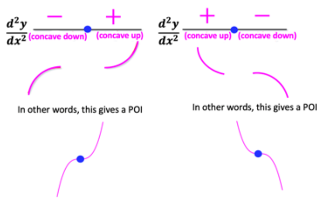
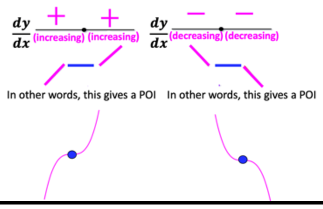
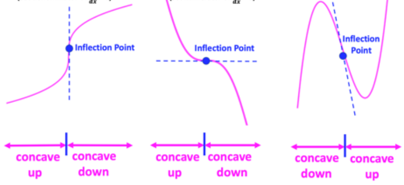
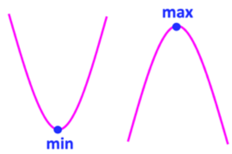
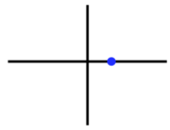
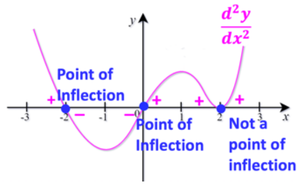
Concave Up Looks Like:

holds water	happy cat	yoga - cow pose
		

Concave Down Looks Like:

spills water	scared cat	yoga - cat pose
		

Terminology	How to find	y graph	$\frac{dy}{dx}$ graph (1st derivative)	$\frac{d^2y}{dx^2}$ graph (2nd derivative)
Increasing  Slope is positive (going upwards when you look at the curve from left to right). Imagine climbing up a hill.	Solve $\frac{dy}{dx} > 0$ This should make sense since the first derivative gives the slope of a function and we know it is positive when increasing. To solve the inequality, remember to solve as in equality first, plot the values found on a number line and then use the sign change test and look for $+$ region.	graph is going upwards (when looked at from left to right)  To help recognize:  tangent lines drawn are increasing	graph is always ABOVE x axis  graph will be in the turquoise region	We can't tell from this graph
Decreasing  Slope is negative (going downwards when you look at the curve from left to right). Imagine sliding down a hill.	Solve $\frac{dy}{dx} < 0$ This should make sense since the first derivative gives the slope of a function and we know it is negative when decreasing. To solve the inequality, remember to solve as in equality first, plot the values found on a number line and then use the sign change test and look for $-$ region.	graph is going downwards (when looked at from left to right)  To help recognize  tangent lines drawn are decreasing	graph is always BELOW x axis  graph will be in the turquoise region	We can't tell from this graph
Stationary Points/ Turning Points/ Max/Min Stationary and turning points are either maximums or minimums. They occur where the graph changes from increasing to decreasing or vice versa.  or  They occur when the slope is zero (since a horizontal line has zero slope)	Solve $\frac{dy}{dx} = 0$ This should make sense since the first derivative gives the slope of a function and we know the slope is equal to zero when we have a stationary/turning point. To classify whether the stationary/turning points are max or min: **Way 1:** use sign of $\frac{d^2y}{dx^2}$ Plug the x value found into $\frac{d^2y}{dx^2}$. If $\frac{d^2y}{dx^2} > 0 \Longrightarrow$ min $\frac{d^2y}{dx^2} < 0 \Longrightarrow$ max **Way 2:** $\frac{dy}{dx}$ sign change test Plug values either side of the value of x found into $\frac{dy}{dx}$. If $\frac{dy}{dx}$ changes from $-$ to $+ \Longrightarrow$ min $+$ to $- \Longrightarrow$ max  	 Note: when $\frac{dy}{dx}$ is undefined max/min will look like sharp turns (corners/nodes and cusps)  How would we find these points? $\frac{dy}{dx}$ is undefined when the derivative is a fraction and the denominator is equal to zero.	zeros (points on the x axis)  In other words  Min: negative to positive slope Max: positive to negative slope Note: $\frac{dy}{dx} = 0$ doesn't guarantee a max/min (i.e. we can have neither like on the graph above). There must be a sign change of $\frac{dy}{dx}$ (this means + to – or – to +) in order to have a max or min.	We can't tell from this graph
Concave Up (aka convex)  If water was poured on the curve, the curve would hold the water	Solve $\frac{d^2y}{dx^2} > 0$ This should make sense to use the second derivative now, since the first derivative talks about the slope and the second derivative talks about concavity which is positive here. To solve the inequality, remember to solve as in equality first, plot the values found on a number line and then use the sign change test and look for $+$ region.	graph looks like the following:  Another way to help recognize:  tangent lines drawn always lie below Note: It should now make sense why $\frac{d^2y}{dx^2} > 0 \Longrightarrow$ min	graph is going upwards  	graph is always ABOVE x axis  graph will be in the turquoise region

Concave Down (aka concave)  If water was poured on the curve, the curve would spill the water	Solve $\frac{d^2y}{dx^2} < 0$ This should make sense since the second derivative talks about concavity (as mentioned in the row above) and our concavity is negative here. To solve the inequality, remember to solve as in equality first, plot the values found on a number line and then use the sign change test and look for $-$ region.	graph looks like the following spills water  scared cat  yoga - cat pose  Another way to help recognize:  tangent lines drawn always lie above Note: It should now make sense why $\frac{d^2y}{dx^2} < 0 \Rightarrow$ max	graph is going downwards 	graph is always BELOW x axis  graph will be in the turquoise region © MyMathsCloud
Points of Inflection (POI)  concavity is zero (the point where the concavity changes i.e. from concave up to down or vice versa)	Solve $\frac{d^2y}{dx^2} = 0$ This should make sense since points of inflection are just where concavity changes to from concave up to down or vice versa. The concavity is neither positive, nor negative and hence it is zero. To prove whether indeed a point on inflection (we need to do this since $\frac{d^2y}{dx^2} = 0$ doesn't <u>guarantee</u> a point of inflection): **Way 1:** Plug the value found into $\frac{d^2y}{dx^2}$. We need a sign change.  Way 2: Plug the value found into $\frac{dy}{dx}$. We need NO sign change. 	graph has a change of concavity There are 3 types of points on inflection **Vertical** **Horizontal** **Slant**  In detail: $\frac{d^2y}{dx^2} = 0$ will pick up all the following 3 types. The value of $\frac{dy}{dx}$ allows us differentiates between the 3 types. **Vertical** $\frac{d^2y}{dx^2} = 0$ AND $\frac{dy}{dx}$ undefined (set denominator of $\frac{dy}{dx} = 0$) **Horizontal** $\frac{d^2y}{dx^2} = 0$ AND $\frac{dy}{dx} = 0$ (set numerator of $\frac{dy}{dx} = 0$) **Slant** $\frac{d^2y}{dx^2} = 0$ AND $\frac{dy}{dx} \neq 0$ 	max or mins 	zeros (points on the x axis)  In other words  There MUST be a change sign (+ to – or – to +) Note: $\frac{d^2y}{dx^2} = 0$ doesn't guarantee a point of inflection, there must be a sign change of $\frac{d^2y}{dx^2}$. © MyMathsCloud